

PREFACE

This text is intended as a broad introduction to some of the various aspects of the field of mathematics known as topology. Topology is one of the younger fields of mathematics. The purpose is for you to learn some of the major ideas and results in the field, to do some hands-on explorations and fairly elementary proofs, and to become aware of some recent questions. It is hoped that you will see mathematics as an active and ever-changing field, with many problems still unsolved. You will also see how various fields of math—algebra, combinatorics, geometry, calculus, and differential equations—interact with topology.

The first three chapters provide the foundation for the course. Chapter 1 is an introduction to some of the basics of point set topology. A solid knowledge of point set topology is essential for most proofs in any topology-related field, but to do a careful and thorough study would consume the entire book. Several fine books are devoted to this study. Since my purpose is to introduce the flavor of topology as a whole, I'll introduce only the bare essentials in the context in which we will work (primarily subsets of Euclidean space). Chapters 2 and 3 introduce surfaces and some of their basic properties. Chapters 4–7 branch out into some of the various other fields of topology: combinatorial topology and map coloring, differential and algebraic topology, and knot theory.

Chapters 1–3 are essential to the rest of the text. However, the remaining chapters are almost completely independent of each other. What do you need as background for this course? A solid theoretical foundation of calculus, certainly. A good understanding of the notions of limits, continuity, and convergence of sequences is essential. A little familiarity with differential equations is assumed in Chapter 5. Some experience with algebraic groups is helpful in Chapter 6 and in a couple of projects in Chapter 7. You will be asked to read and do some proofs, so a basic knowledge of the logic of proofs (use of quantifiers, converse, contrapositive, and proof by contradiction) is assumed. Proofs of almost all of the major theorems we develop and use are given in the text (explicitly or as exercises). In some cases (classification of surfaces in Chapter 2, and Seifert–van Kampen theorem in Chapter 6), these proofs are fairly long and have been separated from the discussion and application of the theorem. You may want to postpone reading the proof until after you have thought about and used the theorem. We have only had to assume a few results without proof.

The text has been used successfully in a capstone course in which the students did all the lecturing, as well as in more traditional lecture-style courses. There are lots of examples and exercises sprinkled throughout the text. They are put there (as opposed to the end of the chapter) to keep you actively involved in the process of learning and discovery. Many are straightforward practice problems and proofs. The more difficult

ones are starred (*), with hints given. The projects are designed to be less routine and are ideal for group work. They may involve some exploratory activities; where there is not a simple answer, they may lead you through more complicated arguments. Be sure to work on them as you read. It really is true that the best way to learn mathematics is to do mathematics. Enjoy.

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