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CORRIGENDUM

p. 23. The regular matrix representation

This representation will not be *simply* isomorphic if there exists an element x of the algebra for which $ax = 0$ for all a of the algebra. The corresponding matrix X would be identically zero. A simply isomorphic representation, however, may be obtained in any case by adjoining a modulus to the algebra before obtaining the regular representation.