

Preface

Character theory provides a powerful tool for proving theorems about finite groups. In fact, there are some important results, such as Frobenius' theorem, for which no proof without characters is known. (Until fairly recently, Burnside's $p^a q^b$ theorem was another outstanding example of this.)

Although a significant part of this book deals with techniques for applying characters to "pure" group theory, an even larger part is devoted to the properties of characters themselves and how these properties reflect and are reflected in the structure of the group.

The reader will need to know some basic finite group theory: the Sylow theorems and how to use them and some elementary properties of permutation groups and solvable and nilpotent groups. A knowledge of additional topics such as transfer and the Schur–Zassenhaus theorem would be helpful at a few points but is not essential. The other prerequisites are Galois theory and some familiarity with rings. In summary, the content of a first-year graduate algebra course should provide sufficient preparation.

Chapter 1 consists of ring theoretic preliminaries, and Chapters 2–6 and 8 contain the basic material of character theory. Chapter 7 is concerned with one of the more important techniques for the application of characters to group theory.

The emphasis in all chapters except 1, 9, 10, and 15 is on characters over the complex numbers rather than on modules and representations over other fields. In Chapter 9, irreducible representations over arbitrary fields are considered; and in Chapter 10, this is specialized to subfields of the complex numbers. Chapter 15 is an introduction (and only that) to Brauer's

theory of blocks and “modular characters” and so is concerned with connections between complex characters and characteristic p -representations.

The remaining chapters concern more specialized topics. Chapter 12 deals with the connections between the set of degrees of the irreducible characters and the structure of a group. Chapter 13 is essentially an exposition of a paper of Glauberman, and Chapter 14 contains some “classical” results on complex linear groups and a small sampling of more recent developments.

Following each chapter there is a selection of problems. In addition to some routine exercises, these include examples, further results, and extensions and variations of theorems in the text. It is hoped that the reader will find that these problems enhance his understanding (and enjoyment) of character theory.