Foreword

Alberto Pedro Calderón was one of the preeminent mathematicians of his time. His fundamental, pioneering work, spanning the second half of the twentieth century, reshaped the landscape of mathematical analysis. We are happy to present in this volume a wide selection from some of Calderón's most influential papers; they range from singular integrals to partial differential equations, from interpolation theory to Cauchy integrals on Lipschitz curves, from inverse problems to ergodic theory. The depth, originality, and historical impact of these works is vividly illustrated by the accompanying multi-faceted commentaries.

We are thankful and greatly indebted to our eminent commentators, who took the time to write their essays specifically for this volume.

We also wish to express our gratitude to the American Mathematical Society for its cooperation and assistance in bringing this project to completion.

Our special thanks go to Eliana Zoque for her excellent and dedicated work in the preparation of this volume for publication.

We hope that future mathematical readers will find this volume enjoyable and informative.

Chicago and Paris
Fall 2007

Alexandra Bellow
Carlos E. Kenig
Paul Malliavin

On becoming a mathematician: markers and decisive moments in Alberto P. Calderón's early life

Alberto Pedro Calderón was born on September 14, 1920, in Mendoza, Argentina, a city at the foot of the Andes. With its strong desert climate, its eternally snow-capped mountains, its picturesque vineyards and olive orchards - where Alberto roamed freely as a child - Mendoza left an indelible imprint on Alberto's imagination. He would return to Mendoza often later in life.

As a child, Alberto was determined to understand how things "really worked". He became good at fixing all sorts of things - mechanical, electrical, or otherwise. As an adult, Alberto was known among family and friends as "the man with the golden hands" because he could fix anything (be it a broken down Olivetti typewriter, a Swiss watch, an out of tune piano); in the process he fabricated the pieces that he needed and came up with the most unorthodox solutions.

Alberto acquired his early love for mathematics and music from his father, a physician, who, realizing his son's unusual gifts, tried to stimulate him. "At the dinner table he would challenge Alberto, a boy of six or seven, to make rapid mental calculations; or, he would play classical music for Alberto and his sister." To the end of his life Alberto could never listen to the Bach Partita in D minor, the Chaconne especially, with anything less than total absorption and rapt emotion. "The link between mathematics and music was there all of his life." This makes one think of Leibniz's famous saying: "Music is the secret arithmetic of the soul, unaware of its act of counting".

There were, however, hurdles on the road to becoming a professional mathematician, "a mathematician's mathematician", as Alberto Calderón was sometimes called, because other mathematicians would come to him for help when they got stuck on a difficult problem.

This essay is based partly on my personal recollections and partly on the talk *"Reminiscencias de mi vida matemática"* given by Alberto P. Calderón at the Universidad Autónoma de Madrid in 1997, upon receiving an Honorary Doctorate, and included in *Discurso de Investidura de Doctor 'Honoris Causa' de los profesores*, printed by the Universidad Autonoma de Madrid, 1997. All quoted material in this essay, not otherwise attributed, is from that source.

"In the beginning, Alberto seemed destined for a career in engineering. His father wanted him to study in the best engineering school in the world, which at the time was ETH (Eidgenössische Technische Hochschule) in Zürich. In order to have him overcome the language barrier and get him ready for ETH, his father sent him, at the age of twelve, to a boarding school in Switzerland, the Montana Knabeninstitut near Zürich. It is here that Alberto met his destiny in the person of Doctor Save Bercovici, the mathematics professor. Their relationship began when Alberto committed a mischievous act in the presence of the professor. The traditional punishment was to send the culprit to his room for three days, during the hours when the boys used to go skiing. But the professor had a different idea. He saw this as a unique opportunity to attract Alberto to mathematics: he gave the boy a problem in Geometry, promising him that if he could solve it, he would be pardoned. The problem was to construct with ruler and compass only, an isosceles triangle, given the height and the sum of the length of the base and one of the sides. With youthful ambition and energy, Alberto set to work and found a construction that solved the problem. Alberto was pardoned, Prof. Bercovici became Alberto's mentor and mathematics moved permanently to the center of Alberto's mental life. Unfortunately, family money ran out, and after two years in Switzerland, Alberto was called back to Argentina."

"Alberto and Dr. Bercovici did not see each other again until thirty years later. Alberto and his first wife, Mabel, were traveling in Europe, they passed through Zürich, and Alberto looked up Dr. Bercovici in the telephone book, finding his name, followed - as was the custom in Switzerland at that time - by a brief description of his professional activities. Dr. Bercovici was a colorful character and in his case the description read: "Mathematics, Physics, Philosophy, Psychotherapy." Dr. Bercovici remembered Alberto, his star pupil, perfectly well. It was a very emotional, somewhat chaotic encounter: the ladies conversed in English, Dr. Bercovici spoke in German and Alberto in French, since Dr. Bercovici understood French but could not speak it, while Alberto understood German but could no longer speak it." Alberto did not tell Save Bercovici about his own academic accomplishments; he was too shy, or too modest, or perhaps he remembered too well how the old Mathematics Professor used to admonish the class: "Boys, please, do not brag! (bitte nicht protzen!)". This was the last time that Save Bercovici and Alberto saw each other.

It is fair to say, however, that the two years that Alberto spent in Switzerland as a schoolboy, were a mind-opening, life-transforming experience that contributed in no small measure to the breadth of intellectual interests and quiet self-confidence that he exhibited all his life. A testimony to this is the following. When Alberto Calderón returned to Chicago from Buenos Aires in 1989, to accept a post-retirement appointment at the University of Chicago and to get married a second time, he came with only three suitcases: one of them contained his favorite papers and books (most of these

books showed signs of wear and tear and had been carefully and lovingly restored by Alberto himself). Here is a partial list:

(1) "A Thousand and One Nights", with illustrations by E. Dulac (upon receiving first prize at the school in Switzerland, 1934)
(2) Will Durant, "The Great Philosophers" (upon receiving first prize at the school in Switzerland, 1934)
(3) J. Rey Pastor, "Análisis Algebraico", Madrid, 1934
(4) Rouché et Comberousse, "Géometrie", Paris, 1900
(5) A. Zygmund, "Trigonometrical Series", First Edition, Warsaw, 1935
(6) de la Vallée Poussin, "Cours d'Analyse Infinitésimale", Paris, 1937
(7) J. Marcinkiewicz, "Collected papers", Warsaw. 1964
(8) S. Saks and A. Zygmund, "Analytic Functions", Warsaw, 1965
(9) G. H. Hardy, "Collected papers", Oxford, 1967
(10) A book of tangos (the lyrics of the best known Argentine tangos). Alberto knew most of these tangos by heart. At home, when listening to tango music, after a glass of wine - preferably a good Malbec from Mendoza - he would sing along or get into the step, for he was a real *"tanguero"*, the tango was in his blood.
(11) Alberto's translation into English of several beloved poems from world literature (Alberto was fluent in several languages and loved poetry). Here is the translation into English, done by Alberto in the spring of 1989, of one of the best known poems of the great Spanish poet Gustavo Adolfo Becquer (Rimas LIII, 938):

Again will dark swallows
hang their nest from your balcony,
and with their black wings
playfully call by your windowpane.
But those which slowed their flight
to contemplate your beauty and my bliss,
the ones that learned our names,
those won't come again.

Again will vines and ivy
climb the walls of your garden
and perhaps bloom in spring
more beautifully than before.
But the dark leaves
covered with crystal drops of dew
we watched tremble, roll and fall
like tears of the day,
those leaves won't grow again.

Again will your ears
hear passionate words of love,
and perhaps your heart
will wake from its slumber.
But, silent and ecstatic
as one prays to God in church,
the way I loved you, have no illusions
you won't be loved that way again.

"Alberto finished high school in Mendoza. Persuaded by his father that he could not make a living as a mathematician, Alberto entered the University of Buenos Aires, studied engineering and graduated as a civil engineer. But being constant and persistent in his affections, he never abandoned mathematics - his great love. Having discovered the "Boletín Matemático Argentino" in the library of his high school in Mendoza, Alberto, once in Buenos Aires, made the acquaintance of its editor, Dr. Bernardo Baidaff." (Later Alberto liked to joke that the Romanians had played a crucial role in his life: the first one was Dr. Save Bercovici, the second one was Dr. Bernardo Baidaff, and the third one was his second wife, myself...). "Dr. Baidaff generously offered Alberto the use of his library and every assistance possible and this led to a lasting friendship. While studying engineering, Alberto established close contacts with the mathematicians at the University of Buenos Aires: he attended the advanced calculus classes of Julio Rey Pastor (the only Professor in the Institute of Mathematics), he participated in a seminar where he became acquainted with the brilliant young Spanish refugees Luis Santaló and Manuel Balanzat. He also met Rey Pastor's assistant, the Argentine mathematician Alberto González Domínguez, a man of vast humanistic culture, who had left behind Greek, Latin, and philology for the sake of mathematics, and who became Alberto's mentor, protector and devoted friend."

"Alberto's wish to become independent, to stop being a financial burden to his father, led him, after graduating as a civil engineer, to the state-owned oil company, the YPF (Yacimientos Petrolíferos Fiscales), where he got a job in the research laboratory of the geophysical division. The job suited him well: he had to work on challenging, difficult problems in applied mathematics, having to do with the design and use of instruments for geophysical prospecting. This was in line with Alberto's mode of thinking, for he firmly believed that mathematics is the Queen of Sciences, and that any good queen has to serve her subjects well." It was in this Lab, in fact, that Alberto conceived of the possibility of determining the conductivity of a body by making electrical measurements at the boundary; he did not publish his results until several decades later, in 1980, in his short Brasilian paper, which pioneered

a whole new area of mathematical research on "inverse problems."[1] Alberto's independence of spirit and exceptional performance, however, annoyed the lab director, who became enraged when he discovered that, in his spare time, Alberto was *passionately* reading Kuratowski's *"Topologie."* "Alberto finally, and very reluctantly, resigned. It was a blessing in disguise, Alberto used to say later, that his lab director made life difficult for him, for otherwise Alberto, who enjoyed his work, would have very likely remained there for the rest of his active life."

As it turned out, this unfortunate episode fortunately led to Alberto becoming a professional mathematician and to his spectacular mathematical career and fame. "Upon resigning his job at the YPF, he got an appointment at the Institute of Mathematics of the University. Soon after that, when Antoni Zygmund, the famous analyst from the University of Chicago, came to visit the University of Buenos Aires in 1948, Alberto was automatically assigned to Zygmund as his assistant." A great mathematical talent was waiting to be "discovered." The rest of the story is, if not history, certainly mathematical history, and is eloquently told in the next chapter.

As Alberto's wife, I shared with him the last nine years of his life (his last Chicago period) and worked with him too, so perhaps some details about Alberto's style of doing mathematics would be in order:

It was sheer joy to discuss a mathematical problem with Alberto, for his mind never traveled the beaten path and could move with great ease from a concrete problem to the most sophisticated abstract setting. He seldom read other mathematicians' papers or books. If he learnt of a new theorem that interested him, he would try to prove it by himself and the techniques that he developed in the process would often have unexpected, far-reaching consequences elsewhere. This was the search for knowledge at its most authentic and inspiring.

Alberto seemed to be oblivious to mathematical prejudices and fads. He was simple and reserved in his manner; he treated colleagues and students alike with kindness and respect. His openness and generosity in sharing mathematical ideas were legendary.

Alberto Calderón had strong emotional and mathematical ties with his native country, Argentina, and with Spain as well, but his enduring "mathematical home" was undoubtedly the University of Chicago, an institution he respected, admired and unequivocally loved all his life long.

The year we were married, 1989, turned out to be a kind of "International Calderón Festival," for Alberto received a lot of recognition that year:

[1]Research on the *"Calderón inverse problem"* is now a vast mathematical enterprise, with important centers in Seattle - University of Washington, Helsinki - R. Nevanlinna Institute, University of Helsinki, Toronto - University of Toronto, to name only some of the most prominent; see also the Commentary by G. Uhlmann in this volume. The Inverse Problems International Association (IPIA) has instituted in 2007 the *Calderón Prize*. This Prize, named in honor of Alberto P. Calderón, is awarded to a "researcher who has made distinguished contributions to the field of inverse problems broadly defined."

the Wolf Prize (from Israel), the Premio de Consagración Nacional (from Argentina), the Steele Prize (from the American Mathematical Society). What an unusual "dowry" to bring to a marriage - friends mused. I knew that Alberto regarded society's insatiable appetite for fame and immortality with benign amusement and a touch of irony. When I marveled at how he could remain so unassuming despite all the acclaim, he would simply answer "I know how little I know". It was the answer of a mathematician's mathematician.[2]

Alexandra Bellow Calderón

[2]I am indebted to my friends Ellen Wright and Leonard Linsky for the invaluable help they gave me in editing this essay.

Alberto P. Calderón the mathematician, his life and works

In the years immediately after World War II, the US Department of State had a very active visitors program that sent prominent scientists to Latin America. Thus, Adrian Albert, Marshall Stone and George Birkhoff visited Buenos Aires, and González Domínguez arranged through them the visit of Antoni Zygmund, whose work on Fourier series he much admired (through his studies with Tamarkin at Brown University, González Domínguez had acquired a strong interest in Fourier Analysis by reading Zygmund's 1935 fundamental treatise, "Trigonometrical Series").

At the Institute of Mathematics, Zygmund gave a two-month seminar on topics in analysis, based on his book. This seminar was attended by González Domínguez, Calderón, Mischa Cotlar and three other young Argentine mathematicians. Each of the participants had to discuss a portion of the text. Calderón's assignment was to present the Marcel Riesz theorem on the continuity of the Hilbert transform in L^p. According to Cotlar's vivid recollection of the event, Calderón's exposition was entirely acceptable to the junior audience but not to Zygmund, who appeared agitated and grimaced throughout. Finally, he abruptly interrupted Calderón to ask where had he read the material he was presenting, and a bewildered Calderón replied that he had read it in Zygmund's book. Zygmund vehemently informed the audience that this was not the proof in his book, and after the lecture took Calderón aside and quizzed him about the new short and elegant proof. Calderón confessed that he had first tried to prove the theorem by himself, and then thinking he could not do it, had read the beginning of the proof in the book; but after the first couple of lines, instead of turning the page, had figured out how the proof would finish... Zygmund recognized Calderón's power, and then and there decided to invite him to Chicago to study with him.

This anecdote illustrates one of Calderón main characteristics: he always sought his own proofs, developed his own methods. From the start, Calderón worked in mathematics that way: he rarely read the work of others farther than the statements of theorems, and after grasping the general

---

This essay, published by arrangement with the University of Chicago Press, is a modified version of the Introduction to the volume "Harmonic Analysis and Partial Differential Equations", *Essays in Honor of Alberto P. Calderón*, University of Chicago Press, 1999.

nature of the problem, went ahead by himself. In this process, Calderón not only rediscovered results, but added new insights to the subject. According to Cotlar, while still in Buenos Aires, Calderón had arrived by himself at something very close to the notion of distributions. This came as a consequence of his interest in quantum mechanics, which had been sparked by Guido Beck, the Austrian-born physicist exiled in Argentina.

The two month visit of Zygmund to Buenos Aires resulted in two joint papers, and an invitation to Calderón to work with him in Chicago. Calderón arrived there in 1949, as a Rockefeller Fellow. The transition was hard, because Calderón felt awed when suddenly confronted with the world of top class mathematics. At the time, the Department of Mathematics at the University of Chicago was reputedly the world's best, and its faculty included Adrian Albert, Shiing S. Chern, Lawrence Graves, Saunders McLane, Marshall Stone (chairman), André Weil, and Antoni Zygmund as professors, with Paul Halmos, Irving Kaplansky, Irving Segal and Edwin Spanier, as assistant professors. Calderón was so shocked that he wanted to return to Argentina, and it required all of Zygmund's power to persuade him to stay in Chicago. Calderón's interests were very broad from the start. Calderón loved all mathematics, and his penchant for concrete analysis did not inhibit his fascination with abstraction. He was interested in the Bourbaki approach, and asked André Weil for open problems. Calderón later said that these problems eventually sparked some of his ideas on algebras of singular integral operators.

As a Rockefeller Fellow, Calderón had gone to Chicago to work with Zygmund, not in pursuit of a degree. But the intervention of Marshall Stone, (a most visionary chairman), encouraged him to obtain a doctorate, without which Calderón's academic career would have been hindered. To make this possible, Stone persuaded him to "staple together" three separate papers into a dissertation. Thus Calderón was able to obtain his Ph.D. in Mathematics under Zygmund's supervision in 1950, only a year after arriving in Chicago. The dissertation proved momentous: each of the three papers solved a long-standing open problem in ergodic theory or harmonic analysis. The two on the behavior of harmonic functions at the boundary opened up the possibility of bypassing complex methods in dealing with fundamental questions of harmonic analysis in the circle, thus leading to the extensions to Euclidean n-dimensional space that were at the core of Zygmund's program for the future.

The collaboration begun by Zygmund and Calderón in 1948 reached fruition in the *Calderón-Zygmund Theory of Singular Integrals*, and lasted more than three decades. This legendary collaboration is reminiscent of the famous Hardy-Littlewood collaboration of the earlier part of the century, but with the added, typically American feature, that the protagonists in this case were brilliant immigrants from different parts of the world. The Calderón-Zygmund memoir *"On singular integrals,"* published in Acta Mathematica in 1952, continues to be one of the most influential papers in

the modern history of analysis. Teacher and student not only forged a major mathematical theory, but founded what became internationally known as the *"Calderón-Zygmund School of Analysis"* (or the *Chicago School of Hard Analysis*). Their methods have far-reaching consequences in many different branches of mathematics. A prime example of such a general method is one of their first joint results, the famous Calderón-Zygmund decomposition lemma, invented to prove the "weak type" continuity of singular integrals of integrable functions, which is now widely used throughout analysis and probability theory. The *Calderón-Zygmund Seminar* has been for several decades (and continues to be) an important tradition in Mathematics at the University of Chicago. It has always met on Mondays at 3:45 in Eckhart Hall #308 and has attracted many gifted graduate students, junior faculty, postdocs as well as mathematicians from all over the Chicago area. After the one hour, state of the art, formal talk, there was, in the old days, a second, informal hour, in which open questions were raised, debated and often solved. The Calderón-Zygmund Seminar has helped shape the development of generations of mathematicians.

Although the theory of singular integrals seemed at first quite arcane, by the mid sixties its popularity was established thanks to the epoch-making contributions to the theory of differential equations made by Calderón using singular integral operators. His proof of the *uniqueness in the Cauchy problem*, using algebras of singular integral operators, his existence and uniqueness theory for hyperbolic problems, his reduction of elliptic boundary value problems to singular integral equations on the boundary (*the method of the Calderón projector*) and the crucial role played by algebras of singular integrals (through the work of Calderón's student R. Seeley) in the *proof of the Atiyah-Singer index theorem*, gained Calderón international fame. The development of pseudo-differential operators, by Kohn-Nirenberg and Hörmander, has its roots in Calderón's work on the applications of algebras of singular integrals to partial differential equations. Indeed, classical pseudo-differential operators form an algebra which includes variable coefficient singular integral operators with kernels infinitely differentiable outside the diagonal, and partial differential operators with smooth coefficients. Calderón himself, in part through his collaborations with R. Vaillancourt and J. Álvarez-Alonso, contributed significantly to the theory of pseudo-differential operators. Nevertheless, Calderón insisted that the earlier point of view of algebras of singular integral operators with non-smooth kernels should provide a tool to solve actual problems arising in physics and engineering, in which lack of smoothness is a natural feature. Moreover, he saw this greater generality as a means to "prepare the ground for applications to the theory of quasi-linear and nonlinear differential operators". This point of view led to what is now referred to as the *"Calderón program"*, whose first success was Calderón's proof of the *boundedness of the "first commutator"*. Further impetus in this program came from Calderón's seminal study of the *Cauchy integral on Lipschitz curves* (1978). Many important works,

like those of Coifman-McIntosh-Meyer, Bony, David-Journé, Verchota and many others, followed from this.

It should not be forgotten that Calderón retained his interest in the applications of mathematics all his life, and some items in his bibliography stem from that interest, e.g. his papers on the phase problem for three-dimensional Fourier expansions, and on the Radon transform. He was thrilled by his work's impact on applications such as signal processing, geophysics, and tomography, and he was intrigued by its relationship to wavelet theory.

After receiving his Ph.D. in Mathematics from the University of Chicago, Calderón taught at Ohio State University for two years. He was a member of the Institute for Advanced Study at Princeton from 1953 to 1955, and an associate professor at MIT from 1955 to 1959, returning to the University of Chicago as a professor from 1959 to 1972, and, after three more years at MIT, as a University Professor of Mathematics from 1975 until his retirement in 1985. He served as Chairman of the Department for two years in the seventies, and returned to it permanently in 1989. Calderón was a Honorary Professor at the University of Buenos Aires from 1975, and the Director of the Instituto Argentino de Matemática (IAM) for several years during the seventies and eighties.

Calderón always maintained close ties with Argentina, and strongly influenced the development of mathematics in his native country. In particular, seven of the sixteen Ph.D. students he supervised at the University of Chicago came from the University of Buenos Aires. This happened in great measure through Zygmund's initiative. As a witness of the extraordinary growth of mathematics in his native Poland during its twenty years of independence between the two World Wars, Zygmund was a strong advocate of developing local mathematical talent throughout the world. He argued that mathematics, needing no major investment in either buildings or laboratories, could initiate the uplifting of a whole scientific establishment. On his 1948 trip to Buenos Aires when he *"discovered"* Calderón, Zygmund also found a group of eager young researchers, among them Mischa Cotlar. After Cotlar had also been *"discovered"* by George Birkhoff, Marshall Stone intervened to have him admitted to the University of Chicago doctoral program, even though he had no prior schooling. He became Zygmund's next Ph.D. student after Calderón. It was Zygmund's second visit to Buenos Aires in 1959 that started a series of visits there by Calderón, and it was Zygmund who personally encouraged a group of undergraduate students and young researchers to attend the University of Chicago to work with Calderón. The first to come were Agnes Benedek and Rafael Panzone (who already had obtained their Ph.D. degrees with González Domínguez and Cotlar, respectively) who developed with Calderón the theory of Banach space valued singular integrals, and Steven Vagi and Evelio Oklander, who became Calderón's Ph.D. students at the University of Chicago. More students followed, and, as was mentioned earlier, all in all seven students

from the University of Buenos Aires obtained their Ph.D. degrees under Calderón's supervision at the University of Chicago.

Calderón and Zygmund also took an active interest in the development of mathematics in Spain. The democracy which emerged in Spain after the era of Franco allowed the blooming of a Spanish school of real analysis which sprang from the activities of the lone Calderón-Zygmund student from Madrid, Miguel de Guzmán, a lifelong friend of both. Miguel had the good fortune - which eluded his Argentine classmates - to return to his country at a time when it flourished, and his energy and generous vision opened the door for Spain to assume a major role on the international scene of mathematical analysis.

Calderón was recognized all over the world for his outstanding contributions to mathematics. He was a member of the U.S. National Academy of Sciences, and of the National Academies of Argentina, Spain and France (corresponding), of the Latin American Academy of Sciences, of the Academy of Sciences of the Third World, and of the American Academy of Arts and Sciences. He received honorary doctorates from the University of Buenos Aires, the Technion (Israel), the Ohio State University, and the Universidad Autónoma de Madrid. He gave many invited addresses to universities and learned societies, and he addressed the International Congress of Mathematicians as invited lecturer in Moscow in 1966 and as plenary lecturer in Helsinki in 1978. He was awarded the 1979 Bôcher Prize, and the 1989 Steele Prize by the American Mathematical Society. He was also the recipient of the Wolf Prize in Mathematics (Israel, 1989) and the Premio de Consagración Nacional (Argentina, 1989).

In 1991, Calderón was honored with the National Medal of Science, the highest award bestowed by the United States for scientific achievement. In that exceptional recognition he followed his teacher Antoni Zygmund, who had received the Medal in 1986. This national recognition of the Calderón-Zygmund School testifies to the enormous capacity of the United States both to integrate immigrants and to benefit from their contributions.

Besides his remarkable research accomplishments, Calderón was a gifted lecturer and an inspiring, though uneven, teacher; he could teach magnificent courses, but could also lecture obscurely, depending on the degree of his current involvement with the subject. Calderón supervised twenty seven Ph.D. students: five while at MIT, sixteen at the University of Chicago and six at the University of Buenos Aires. His students and collaborators had the opportunity to work with a formidable mathematician, with an array of hard and interesting problems to suggest, and all profited from his unusual openness in sharing ideas.

Outside of scientific endeavors, Calderón's interest ranged widely. He was fluent in several languages, loved music, played the piano, and danced the tango beautifully. He had a natural talent at fixing all sort of appliances, and always retained the mechanical interests of his early years. To each of his activities he brought the same enthusiasm and the same fresh outlook.

In 1950, Calderón married Mabel Molinelli Wells, a mathematics graduate whom he had met while both were students at the University of Buenos Aires, and who was to have a very strong influence on his life. They had a daughter, María Josefina (Mary Jo), and a son, Pablo. In the late seventies Calderón's family settled in Buenos Aires, where Mabel died in 1985 after a long illness. Mary Jo Calderón now lives in Paris. Pablo Calderón resides in Connecticut with his wife and their three children. Another mathematician in the family, Calixto P. Calderón, lives in the Chicago area; as a tribute to his older brother, he published a posthumous joint paper with Alberto Calderón.

During a visit to MIT, Calderón had shared an office with the Romanian-born mathematician Alexandra (Bagdasar) Bellow, an ergodic theorist of world renown, currently Emeritus Professor at Northwestern University. In 1989 Alexandra and Alberto Calderón were married. At the end of her 1991 Noether Lecture, Alexandra described their shared personal and professional fulfillment, saying "life offered (me) peace and happiness after fifty, and mathematics became an asset in the human aspects of my personal life." Calderón had been interested in ergodic theory throughout his career, and his last two papers are joint with his wife.

Alberto P. Calderón, Emeritus Professor of Mathematics of the University of Chicago and Honorary Professor of the University of Buenos Aires, died at the age of 77 on April 16, 1998, after a short illness. He was one of the greatest mathematicians of the second half of the twentieth century.

Michael Christ
Carlos E. Kenig
Cora Sadosky

Curriculum Vitae

Born in Mendoza, Argentina, September 14, 1920
Died in Chicago, USA, April 16, 1998

Education

Civil Engineering Degree, University of Buenos Aires, 1947
Doctor of Philosophy in Mathematics, University of Chicago, 1950

Research, teaching and visiting positions

1949 -1950 Rockefeller Foundation Fellow, University of Chicago
1950 -1953 Visiting Associate Professor, Ohio State University, Columbus, Ohio
1953 -1955 Member, Institute for Advanced Study, Princeton, New Jersey
1955 -1959 Associate Professor, Massachusetts Institute of Technology
1959 -1968 Professor, University of Chicago
1972-1975 Professor, Massachusetts Institute of Technology
Visiting Professor at various times at the following universities: University of Buenos Aires, Cornell University, Stanford University, National University of Bogotá, Colombia, Collège de France, Paris, University of Paris (Sorbonne), Autónoma and Complutense Universities, Madrid, University of Rome, Göttingen University

Named Professorships

1968 -1972, *Louis Block Professor of Mathematics*, University of Chicago
1975 -1985, *University Professor of Mathematics*, University of Chicago
1975 - , *Honorary Professor*, University of Buenos Aires

Honors, academies

1958 Member, American Academy of Arts and Sciences, Boston, Massachusetts
1959 Correspondent Member, National Academy of Exact, Physical and Natural Sciences, Buenos Aires, Argentina
1968 Member, National Academy of Sciences of the U.S.A.
1970 Correspondent Member, Royal Academy of Sciences, Madrid, Spain
1983 Member, Latin American Academy of Sciences
1984 Member, National Academy of Exact, Physical and Natural Sciences, Buenos Aires, Argentina

1984 Foreign Associate, Institut de France, Paris, France
1984 Member, Third World Academy of Sciences, Trieste, Italy

Honors, prizes

1969 *Latin American Prize in Mathematics*, Awarded by IPCLAR (Instituto para la Promoción de las Ciencias, Letras y Realizaciones), Santa Fe, Argentina
1979 *Bôcher Memorial Prize*, awarded by the American Mathematical Society
1983 *Konex Prize (Science and Technology)*, Buenos Aires, Argentina
1989 *Premio de Consagración Nacional*, Buenos Aires, Argentina
1989 *Wolf Prize*, awarded by the Wolf Foundation, Jerusalem, Israel
1989 *Steele Prize*, awarded by the American Mathematical Society
1991 *National Medal of Science*, Washington, U.S.A

Honorary Degrees

Doctor Honoris Causa, University of Buenos Aires, Argentina, 1969.
Doctor of Science, Honoris Causa, Technion, Haifa, Israel, 1989.
Doctor of Science, Honoris Causa, Ohio State University, 1995.
Doctor Honoris Causa, Universidad Autónoma de Madrid, Spain, 1977.

Ph.D. students of Alberto P. Calderón

Robert T. Seeley (1958)
Irwin S. Bernstein (1959)
I. Norman Katz (1959)
Jerome H. Neuwirth (1959)
Earl Robert Berkson (1960)
Evelio Oklander (1964)
Cora S. Sadosky (1965)
Stephen Vagi (1965)
Umberto Neri (1966)
John C. Polking (1966)
Nestor Marcelo Rivière (1966)
Carlos Segovia Fernández (1967)
Miguel de Guzmán (1968)
Daniel Fife (1968)
Alberto Torchinsky (1971)
Keith W. Powls (1972)
Josefina Dolores Álvarez Alonso (1976)
Telma Caputti (1976)
Robert Richard Reitano (1976)
Carlos E. Kenig (1978)
Ángel Bartolomé Gatto (1979)
Cristián Enrique Gutierrez (1979)

Kent G. Merryfield (1980)
Michael Christ (1982)
Gerald M. Cohen (1982)
María Amelia Muschietti (1984)
Marta Susana Urciolo (1985)

Publications of A. P. Calderón

[1] Calderón, A. P., González Domínguez, A., and Zygmund, A., "Nota sobre los valores límites de funciones analíticas", *Revista de la Unión Matemática Argentina*, **14** (1949), pp. 16–19.

[2] Calderón, A. P., "On theorems of M. Riesz and A. Zygmund", *Proc. Amer. Math. Soc.*, **1** (1950), pp. 533–535.

[3] Calderón, A. P., "On the behavior of harmonic functions at the boundary", *Trans. Amer. Math. Soc.*, **68** (1950), pp. 47–54.

[4] Calderón, A. P. and Zygmund, A., "Note on the boundary values of functions of several complex variables", Contributions to Fourier Analysis, *Annals of Math. Studies*, **25** (1950), Princeton Univ. Press, Princeton, N. J., pp. 145–165.

[5] Calderón, A. P. and Zygmund, A., "On the theorem of Hausdorff-Young and its extensions", Contributions to Fourier Analysis, *Annals of Math. Studies*, **25** (1950), Princeton Univ. Press, Princeton, N. J., pp. 166–188.

[6] Calderón, A. P., "On a theorem of Marcinkiewicz and Zygmund", *Trans. Amer. Math. Soc.*, **68** (1950), pp. 55–61.

[7] Calderón, A. P. and Zygmund, A., "On singular integrals in the theory of the potential", *Proc. Int. Congress of Math.*, **1** (1950).

[8] Calderón, A. P.,"On the differentiability of absolutely continuous functions", *Revista Mat. Univ. di Parma*, **2** (1951), pp. 203–213.

[9] Calderón, A. P. and Zygmund, A., "Note on the interpolation of linear operations", *Studia Math.*, **12** (1951), pp. 194–204.

[10] Calderón, A. P. and Klein, G., "On an extremum problem concerning trigonometrical polynomials", *Studia Math.*, **12** (1951), pp. 166–169.

[11] Calderón, A. P. and Zygmund, A., "On the existence of certain singular integrals", *Acta. Math.*, **88** (1952), pp. 85–139.

[12] Calderón, A. P. and Pepinsky, R., *"On the phases of Fourier coefficients of positive real periodic functions, Computing methods and the phase problem in X-ray crystal Analysis"*, Dept. of Physics, Penn. State College (1952), pp. 339–349.

[13] Calderón, A. P., "A general ergodic theorem", *Annals of Math.*, (2) **58** (1953), pp. 183–191.

[14] Calderón, A. P. and Mann, H. B., "On the moments of stochastic integrals", *Sankhya*, **12** (1953), pp. 347–350.
[15] Calderón, A. P., *"Solution of the scalar radiation problem for surfaces of revolution"*, Ohio State Univ. Research Foundation, (1953), Antenna Laboratory, A. F. 18 (600) 88, 478–17, 4 pp.
[16] Calderón, A. P. and Arens, R., *"Analytic functions of Fourier transforms"*, Segundo Simposio sobre algunos problemas matemáticos que se están estudiando en Latinoamérica. Centro de la UNESCO de Cooperación Científica para América Latina, Montevideo, Uruguay (1954), pp. 39–52.
[17] Calderón, A. P., *"Singular integrals"*, Segundo Simposio sobre algunos problemas matemáticos que están estudiando en Latinoamérica. Centro de la UNESCO de Cooperación Científica para América Latina, Montevideo, Uruguay (1954), pp. 319–328.
[18] Calderón, A. P., "The multipole expansion of radiation fields", *Journal of Rational Mech. Anal.*, **3** (1954), pp. 523–537.
[19] Calderón, A. P. and Zygmund, A., "Singular integrals and periodic functions", *Studia Math.*, **14** (1954), pp. 249–271.
[20] Calderón, A. P. and Zygmund, A., "On a problem of Mihlin", *Trans. Amer. Math. Soc.*, **78** (1955), pp. 209–224.
[21] Arens, R. and Calderón, A. P., "Analytic functions of several Banach Algebra elements", *Annals of Math. (2)*, **62** (1955), pp. 204–216.
[22] Calderón, A. P., "Sur les mesures invariantes", *C. R. Acad. Sci. Paris*, **240** (1955), pp. 1960–1962.
[23] Calderón, A. P. and Devinatz, A., "On the Fourier-Stieltjes transforms", *Canad. J. Math*, **7** (1955), pp. 453–461.
[24] Calderón, A. P. and Devinatz, A., "Sur certaines courbes dans l'espace de Hilbert", *C. R. Acad. Sci. Paris*, **241** (1955), pp. 539–541.
[25] Calderón, A. P. and Devinatz, A., "Sur certaines courbes á courbure constante dans l'espace de Hilbert", *C. R. Acad. Sci. Paris*, **241** (1955), pp. 586–587.
[26] Calderón, A. P. and Zygmund, A., "A note on the interpolation of sublinear operations", *Amer. J. Math.*, **78** (1956), pp. 282–288.
[27] Calderón, A. P. and Zygmund, A., "On singular integrals", *Amer. J. Math.*, **78** (1956), pp. 289–309.
[28] Calderón, A. P., *"Ideals in Abelian group algebras"*, Symposium on Harmonic Analysis and related integral transforms, Technical Report, Dept. of Math., Cornell Univ., Ithaca, N. Y., (1956), 12 pp.
[29] Calderón, A. P. and Zygmund, A., "Algebras of certain singular integral operators", *Amer. J. Math.*, **78** (1956), pp. 310–320.

[30] Calderón, A. P. and Zygmund, A., "Singular integrals operators and differential equations", *Amer. J. Math.*, **79** (1957), pp. 901–921.
[31] Calderón, A. P. and Zygmund, A., "On a problem of Mihlin", *Trans Amer. Math. Soc.*, **84** (1957), pp. 559–560. Addenda to the paper.
[32] Calderón, A. P., "Uniqueness in the Cauchy problem for partial differential equations", *Amer. J, Math.*, **80** (1958), pp. 16–36.
[33] Calderón. A. P, Spitzer, F and Widom, H., "Inversion of Toeplitz matrices", *Illinios J. Math.*, **3** (1959), pp. 490–498.
[34] Calderón, A. P., *Integrales singulares y sus aplicaciones a ecuaciones diferenciales hiperbólicas*, Cursos y Seminarios de Matemática, Fasc. **3** (1960), Universidad de Buenos Aires, 121 pp.
[35] Calderón, A. P. and Zygmund, A., "A note on local properties of solutions of elliptic differential operators", Proc. Nat. Acad. Sci. U.S.A, **46** (1960), pp. 1385–1389.
[36] Calderón, A. P. and Zygmund, A., "On the differentiability of functions which are of bounded variation in Tonelli's sense", *Revista Unión Mat. Arg.*, **20** (1960), pp. 102–121.
[37] Calderón, A. P., "Lebesgue spaces of differentiable functions and distributions", *Proc. Symp. Pure Math.* **IV**, (1961), pp. 33–49, Amer. Math. Soc., Providence, Rhode Island.
[38] Calderón, A. P., *Existence and uniqueness theorems for systems of partial differential equations*, Proc. Symp. Fluid Dynamics and Applied Math., Univ. of Maryland, (1961), pp. 147–195, Gordon and Breach, New York.
[39] Calderón, A. P. and Zygmund, A., "Local properties of solutions of elliptic partial differential equations", *Studia Math.*, **20** (1961), pp. 171–225.
[40] Benedek, A., Calderón, A. P. and Panzone, R., "Convolution operators on Banach space valued functions", *Proc. Nat. Acad. Sci.* U.S.A, **48**, No. 3 (1962), pp. 356–365.
[41] Calderón, A. P, "Intermediate spaces and interpolation", *Studia Math., Seria Sepecjalna*, Z. I. (1963), pp. 31–34.
[42] Calderón, A. P, *"Boundary value problems for elliptic equations"*, Outlines of the joint Soviet-American Symposium on Partial Differential Equations", August 1963, pp. 303–304.
[43] Calderón, A. P. and Zygmund, A., "Higher gradients of harmonic functions", *Studia Math.*, **24** (1964), pp. 211–226.
[44] Calderón, A. P, "Intermediate spaces and interpolation, the complex method", *Studia Math.*, **24** (1964), pp. 113–190.
[45] Calderón, A. P.,"Commutators of singular integral operators", *Proc. Nat. Acad. Sci.* U.S.A, **53** (1965), pp. 1092–1099.
[46] Calderón, A. P., "Spaces between L^1 and L^∞ and the theorem of Marcinkiewicz", *Studia Math.*, **26** (1966), pp. 273–299.

[47] Calderón, A. P., "Singular integrals", *Bulletin Amer. Math. Soc.*, **72** (1966), pp. 427-466.
[48] Calderón, A. P., "Estimates for integral operators" A. F. *Office of Sci. Research*, 65-1866 (1966), pp. 3–12.
[49] Calderón, A. P., "Algebras of singular integral operators", *Proc. Symp. Pure Math.*, **10** (1966), pp. 18–55, Amer. Math. Soc., Providence, Rhode Island.
[50] Calderón, A. P., Weiss, M. and Zygmund, A., "On the existence of singular integrals", *Proc. Symp. Pure Math.*, **10** (1966), pp. 56–73, Amer. Math. Soc., Providence, Rhode Island.
[51] Calderón, A. P., "The analytic calculation of the index of elliptic equations", *Proc. Nat. Acad. Sci.* U.S.A, **57** (1967), pp. 1193–1194.
[52] Calderón, A. P., "Ergodic theory and translation invariant operators", *Proc. Nat. Acad. Sci*, U.S.A, **59** (1968), pp. 349–353.
[53] Calderón, A. P., *"A priori estimates for singular integrals operators"*, Centro Internazionale Matematico Estivo (1969).
[54] Calderón, A. P., "Uniqueness of distributions", *Revista Unión Mat. Arg.*, **25** (1970), pp. 37–65.
[55] Calderón, A. P. and Vaillancourt, R., "On the boundedness of pseudo-differential operators", *J. Math Soc. Japan*, **23** (1971), pp. 274–278.
[56] Calderón, A. P. and Valliancourt, R., "A class of bounded pseudo-differential operators", *Proc. Nat. Acad. Sci.* U.S.A, **69** (1972), pp. 1185–1187.
[57] Calderón, A. P., "Estimates for singular integral operators in terms of maximal functions", *Studia Math.*, **44** (1972), pp. 563–581.
[58] Calderón, A. P. and Zygmund, A. "On singular integrals", *Studia Math.*, **46** (1973), pp. 297–299. Addendum to the paper.
[59] Calderón, A. P., "A note on biquadratic forms", *Journal of Linear Algebra and Appl.*, **7** (1973), pp. 175–177.
[60] Calderón, A. P. and Torchinsky, A., "Parabolic maximal functions associated with a distribution", *Advances in Math.*, **16** (1975), pp. 1–63.
[61] Calderón, A. P., *Lecture Notes on Psuedo-Differential Operators and Elliptic Boundary Value Problems.* I Cursos de Matemática, Instituto Argentino de Matemática (1976), Buenos Aires, 83 pp.
[62] Calderón, A. P., "An inequality for integrals", *Studia Math.*, **57** (1976) pp. 275–277.
[63] Calderón, A. P., "On an integral of Marcinkiewicz", *Studia Math.*, **57** (1976), pp. 279–284.
[64] Calderón, A. P., "Inequalities for the maximal function relative to a metric", *Studia Math.*, **57** (1976), pp. 297–306.

[65] Calderón, A. P., "Cauchy integrals on Lipschitz curves and related operators", *Proc. Nat. Acad. Sci. U.S.A*, **74** (1977), pp. 1324–1327.

[66] Calderón, A. P. and Torchinsky, A., "Parabolic maximal functions associated with a distribution II", *Advances in Math.*, **25** (1977), pp. 101–171.

[67] Calderón, A. P., "An atomic decomposition of distributions in parabolic H^p spaces", *Advances in Math.*, **25** (1977), pp.216–225.

[68] Calderón, A. P. and Scott, R., "Sobolev inequalities for $p > 0$", *Studia Math.*, **62** (1978), pp.75–92.

[69] Calderón, A. P.; Calderón, C. P.; Fabes, E.; Jodeit, M. and Rivière, N. M., "Applications of the Cauchy integrals on Lipschitz curves", *Bull. Amer. Math. Soc.*, **84** (1978), pp. 287–290.

[70] Calderón, A. P. and Zygmund, A., "On singular integrals with variable kernels", *Applicable Analysis*, **7** (1978), pp. 221–238, Gordon and Breach Publ., Great Britain.

[71] Calderón, A. P. and Zygmund, A., "A note on singular integrals", *Studia Math.*, **65** (1979), pp. 77–87.

[72] Álvarez Alonso, J. D. and Calderón, A. P., *Functional calculi for pseudo-differential operators, Fourier Analysis,* Proc. Sem., El Escorial, (1979), pp. 3–61, Asociación Matemática Española, Madrid, 1980.

[73] Calderón, A. P., *Commutators, Singular Integrals on Lipschitz Curves and Applications*, Proc. Int. Congress of Mathematicians, (Helinski, 1978), pp. 85–96, Acad. Sci. Fennica, Helinski, 1980.

[74] Calderón, A. P., "On an inverse boundary value problem", Sociedade Brasileria de Matemática, Rio de Janeiro, *Atas*, **12** (1980), pp. 67–73.

[75] Calderón, A. P., *On the Radon Transform and some of its generalizations*, Conference on Harmonic Analysis in honor of Antoni Zygmund, **II** (1981), pp. 673–689, Wadsworth Math. Series.

[76] Calderón, A. P. and Capri, O. N., "On the convergence in L^1 of singular integrals", *Studia Math.*, **78** (1984), pp. 321–327

[77] Álvarez Alonso, J. D. and Calderón, A. P., *Functional calculi for pseudo-differential operators.* II, Studies in Appl. Math., Adv. Math. Suppl. Stud., 8 (1983), pp. 27–72.

[78] Calderón, A. P., "Boundary value problem for the Laplace equation in Lipschitzian domains", *Recent progress in Fourier Analysis (El Escorial, 1983)*, pp. 33-48, North Holland Math. Stud., 111, *North-Holland, Amsterdam*, 1985.

[79] Calderón, A. P., *Integrales singulares y operadores pseudodiferenciales.* Historia y perspectiva, Anales Acad. Nac. Ciencias Exac. Fís. y Nat., Buenos Aires, **38** (1986), pp. 33–45.

[80] Calderón, A. P., "Reflexiones sobre el aprendizaje y enseñanza de la Matemática", *Revista de Educación Matemática*, Unión Matemática Argentina, **3**, No. 1 (1987), pp. 3–13.

[81] Calderón, A. P., *Presentation of Dr. Eduardo Zarantonello as correspondent member of the Academia Nacional de Ciencias Exactas, Físicas y Naturales*, Buenos Aires, Argentina. Annals of the Academia, **40** (1988).

[82] Calderón, A. P., *Presentation of Dr. Misha Cotlar as correspondent member of the Academia Nacional de Ciencias Exactas, Físicas y Naturales*, Buenos Aires, Argentina. Annals of the Academia, **41** (1989).

[83] Calderón, A. P., *Presentation of Dr. Carlos Segovia Fernández as correspondent member of the Academia Nacional de Ciencias Exactas, Físicas y Naturales*, Buenos Aires, Argentina, Annals of the Academia, **42** (1990).

[84] Calderón, A. P. and Sagher, Y., *The Hilbert Transform of the Gaussian,* Almost Everywhere Convergence II (Proc. of the 1989 International Conference on a.e. convergence in Probability and Ergodic Theory, Evanston, Illinois; edited by Alexandra Bellow and Roger L. Jones), pp. 109–112, Academic Press, 1991.

[85] Bellow, A.; Calderón, A. P. and Krengel, U., "Hopf's ergodic theorem for particles with different velocitites and the "strong sweeping out property"", *Canad. Math. Bull.*, **38** (1), (1995), pp. 11–15.

[86] Bellow, A. and Calderón, A. P., *A weak-type inequality for convolution products*, Harmonic Analysis and Partial Differential Equations (Proc. of an international conference held at the Univ. of Chicago in 1996, in honor of Alberto P. Calderón's 75th birthday; edited by Michael Christ, Carlos E. Kenig and Cora Sadosky), pp. 41-48, Univ. of Chicago Press, 1999.

[87] Calderón, A. P. and Calderón, C. P., "A Representation Formula and its Applications to Singular Integrals", *Indiana Univ. Math. J.*, **49** (1), (2000), pp. 1–5.