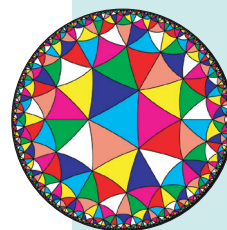


# Contents

## 2 First of Seven Millennium Problems Nears Completion

by Barry Cipra

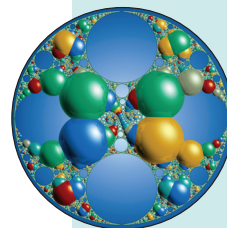
In 2002, Grigory Perelman announced a solution to the Poincaré Conjecture, a problem in topology selected in 2000 as one of the seven leading math challenges of the millennium. After more than three years of scrutiny, mathematicians are cautiously accepting his proof, which uses a geometric partial differential equation called “Ricci flow,” first studied by Richard Hamilton.



## 14 Classifying Hyperbolic Manifolds —All’s Well that Ends Well

by Dana Mackenzie

While three-dimensional topologists awaited a verdict on the Poincaré Conjecture (which deals with closed manifolds), several of them solved a suite of long-standing conjectures about the ends of open manifolds.



## 28 Digits of Pi

by Barry Cipra

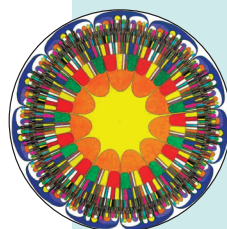
Pi lovers rejoice! You can now stick your finger into pi and pluck out any digit (say, the trillionth one) without having to compute all the preceding digits. The only catch is that you have to count in sixteens.



## 40 Combinatoricists Solve a Venn-erable Problem

by Barry Cipra

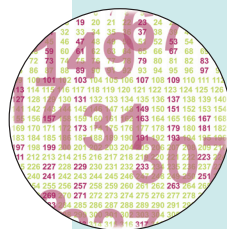
A question posed by an undergraduate student—can a Venn diagram be rotationally symmetric?—leads to some beautiful “doilies,” intricate mathematics, and (after forty years) a solution found by another student.

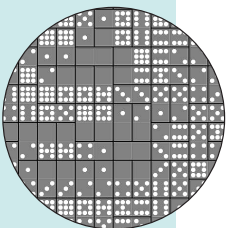
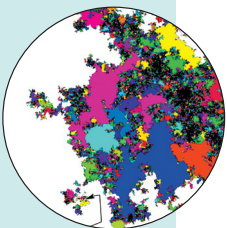
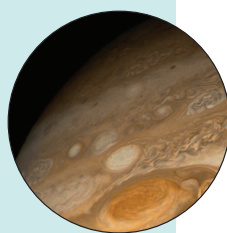
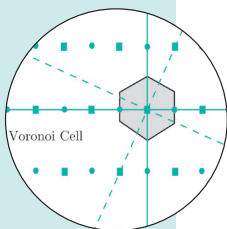


## 52 New Insights into Prime Numbers

by Barry Cipra

Three major discoveries about the distribution of prime numbers surprised number theorists. A team of Indian mathematicians, including two students, discovers the first polynomial-time algorithm for testing to see if a number is prime. American, Turkish, and Hungarian mathematicians collaborate to show that small gaps between consecutive primes occur much more often than anyone had previously proved. And another international collaboration finds evenly spaced (but non-consecutive) prime sequences of any desired length.





## 66 **From Rubik's Cube to Quadratic Number Fields ... and Beyond**

by Dana Mackenzie

Manjul Bhargava started with a simple idea of putting numbers into a box. What came out of the box, eventually, was a whole new way to combine and to count objects in algebraic number theory.

## 78 **Vortices in the Navier-Stokes Equations**

by Barry Cipra

From Jupiter's Great Red Spot to the eddies in a stream, vortices are a familiar feature of fluid flow. Mathematicians have gained new insights into the formation and long-term survival of vortices in both two- and three-dimensional fluids.

## 86 **Fluid Dynamics Explains Mysteries of Insect Motion**

by Dana Mackenzie

How do water striders move on a nearly frictionless surface? How do dragonflies hover? These and other conundrums of biology can be explained by the mathematics of the Navier-Stokes equations.

## 100 **Brownian Motion, Phase Transitions, and Conformal Maps**

by Dana Mackenzie

A new random process called Schramm-Loewner evolution turns out to be a good model for a variety of physical phenomena, from the random jitters of air molecules to phase transitions of a magnetic material. The key mathematical property of these systems, proven in some cases and still conjectural in others, is conformal invariance.

## 112 **Smooth(ed) Moves**

by Barry Cipra

Some algorithms of computer science, such as the simplex algorithm for solving linear programming problems, work better than they are supposed to. A new measure of complexity, called smoothed analysis, shows why the standard worst-case scenarios are so misleading.