

Contents

Introduction	ix
Some general notation and conventions	xiii
Chapter 1. Overview of our main results	1
Part 1. Exposition of background material	5
Chapter 2. Review of selected aspects of group cohomology	7
2.1. Some features of group cohomology via the algebraic approach	7
2.2. Some features of group cohomology via the classifying space	9
2.3. The relation between the coefficient rings \mathbb{Z} and \mathbb{F}_p	12
2.4. The duality relation between homology and cohomology	14
2.5. The Borel construction and G -equivariant homology	16
2.6. Spectral sequences for G -equivariant homology	20
2.7. Review of some aspects of simplicial complexes	23
Chapter 3. Simplicial sets and their equivalence with topological spaces	27
3.1. The context of simplicial sets	27
3.2. A presentation for the category Δ of finite ordered sets	29
3.3. Singular simplices and geometric realization	30
3.4. Products and function spaces	32
3.5. Categorical settings for homotopy theory	35
3.6. Model categories, Kan complexes and Quillen's equivalence	36
3.7. Simplicial R -modules and homology	40
Chapter 4. Bousfield-Kan completions and homotopy colimits	43
4.1. Cosimplicial objects	43
4.2. Bousfield-Kan completions	45
4.3. Completion of the classifying space BG	48
4.4. Simplicial spaces	50
4.5. Diagrams of spaces and homotopy colimits	52
4.6. The homotopy colimit over a simplex category	61
4.7. The Borel construction as a homotopy colimit	65
4.8. The Bousfield-Kan homology spectral sequence	67
Chapter 5. Decompositions and ample collections of p -subgroups	73
5.1. Some history of homology and homotopy decompositions	74
5.2. Homology decompositions defined by homotopy colimits	83
5.3. Ampleness for collections of subgroups	87
5.4. The centralizer decomposition	89

5.5.	The subgroup decomposition	90
5.6.	The normalizer decomposition	92
5.7.	Ampleness for the centric collection	99
5.8.	Sharpness for the three decompositions	100
5.9.	The “standard” ample G -homotopy type defined by $\mathcal{S}_p(G)$	103
5.10.	Ample (but potentially inequivalent) subcollections of $\mathcal{A}_p(G)$	106
5.11.	Ample (but potentially inequivalent) subcollections of $\mathcal{B}_p(G)$	107
Chapter 6.	2-local geometries for simple groups	109
6.1.	Some history of geometries for simple groups	110
6.2.	Preview: some initial examples of local geometries	113
6.3.	Some general constructions of 2-local geometries	126
6.4.	Flag-transitive action on geometries	136
6.5.	Some equivalence methods for 2-local geometries	142
6.6.	Final remarks on 2-local geometries	150
Part 2.	Main results on sporadic groups	153
Chapter 7.	Decompositions for the individual sporadic groups	155
7.1.	The Mathieu group M_{11}	157
7.2.	The Mathieu group M_{12}	161
7.3.	The Mathieu group M_{22}	165
7.4.	The Mathieu group M_{23}	172
7.5.	The Mathieu group M_{24}	178
7.6.	The Janko group J_1	180
7.7.	The Janko group J_2	181
7.8.	The Janko group J_3	183
7.9.	The Janko group J_4	187
7.10.	The Higman–Sims group HS	190
7.11.	The McLaughlin group McL	194
7.12.	The Suzuki group Suz	197
7.13.	The Conway group Co_3	200
7.14.	The Conway group Co_2	203
7.15.	The Conway group Co_1	206
7.16.	The Fischer group Fi_{22}	208
7.17.	The Fischer group Fi_{23}	211
7.18.	The Fischer group Fi'_{24}	215
7.19.	The Harada–Norton group $HN = F_5$	222
7.20.	The Thompson group $Th = F_3$	224
7.21.	The Baby Monster $B = F_2$	226
7.22.	The Fischer–Griess Monster $M = F_1$	229
7.23.	The Held group He	231
7.24.	The Rudvalis group Ru	233
7.25.	The O’Nan group $O’N$	237
7.26.	The Lyons group Ly	242
Chapter 8.	Details of proofs for individual groups	247
8.1.	Details for M_{12}	247
8.2.	Details for J_3	249

8.3. Details for HS	250
8.4. Details for Fi_{22}	253
8.5. Details for Fi'_{24}	255
8.6. Details for HN	264
8.7. Details for Ru	265
8.8. Details for $O'N$	272
Bibliography	275
Index	281