

Contents

Preface	vii
Chapter 1. Introduction	1
1.1. Random polynomials and their zeros	1
1.2. Basic notions and definitions	6
1.3. Hints and solutions	11
Chapter 2. Gaussian Analytic Functions	13
2.1. Complex Gaussian distribution	13
2.2. Gaussian analytic functions	15
2.3. Isometry-invariant zero sets	18
2.4. Distribution of zeros - The first intensity	23
2.5. Intensity of zeros determines the GAF	29
2.6. Notes	31
2.7. Hints and solutions	32
Chapter 3. Joint Intensities	35
3.1. Introduction – Random polynomials	35
3.2. Exponential tail of the number of zeros	37
3.3. Joint intensities for random analytic functions	39
3.4. Joint intensities – The Gaussian case	40
3.5. Fluctuation behaviour of the zeros	42
Chapter 4. Determinantal Point Processes	47
4.1. Motivation	47
4.2. Definitions	48
4.3. Examples of determinantal processes	53
4.4. How to generate determinantal processes	63
4.5. Existence and basic properties	65
4.6. Central limit theorems	72
4.7. Radially symmetric processes on the complex plane	72
4.8. High powers of complex polynomial processes	74
4.9. Permanent processes	75
4.10. Notes	79
4.11. Hints and solutions	80
Chapter 5. The Hyperbolic GAF	83
5.1. A determinantal formula	83
5.2. Law of large numbers	90
5.3. Reconstruction from the zero set	91
5.4. Notes	95

5.5. Hints and solutions	98
Chapter 6. A Determinantal Zoo	99
6.1. Uniform spanning trees	99
6.2. Circular unitary ensemble	100
6.3. Non-normal matrices, Schur decomposition and a change of measure	103
6.4. Ginibre ensemble	105
6.5. Spherical ensemble	106
6.6. Truncated unitary matrices	107
6.7. Singular points of matrix-valued GAFs	112
6.8. Notes	116
Chapter 7. Large Deviations for Zeros	119
7.1. An Offord type estimate	119
7.2. Hole probabilities	121
7.3. Notes	132
Chapter 8. Advanced Topics: Dynamics and Allocation to Random Zeros	135
8.1. Dynamics	135
8.2. Allocation	137
8.3. Notes	144
8.4. Hints and solutions	146
Bibliography	149